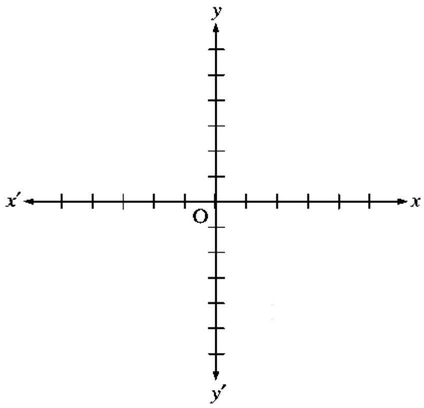
1. Cartesian Coordinates

→ Two perpendicular axes: **X-axis** (horizontal) **Y-axis** (vertical)
→ Their intersection point is called the **Origin O(0,0)**.
→ Coordinates of a point are written as **(x, y)**.
x-coordinate (Abscissa) → distance from y-axis.
y-coordinate (Ordinate) → distance from x-axis.
→ Order is important: **(x, y) ≠ (y, x)**.
→ Quadrants



Quadrant	Sign of x	Sign of y
I	+	+
II	–	+
III	–	–
IV	+	–

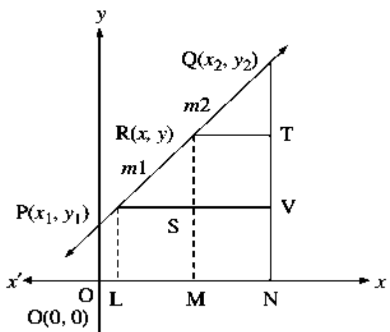
2. Distance Formula

For points $P(x_1, y_1)$and $Q(x_2, y_2)$: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance from origin: $OP = \sqrt{x^2 + y^2}$

Applications:

- Finding length of line segment
- Checking collinearity
- Proving triangles (right, equilateral, etc.)



Derived the distance formula between two points P (x_1 ,y_1) and Q (x_2 ,y_2) in the following manner: Let us draw a line |l XX' through P. Let R be the point of intersection of the perpendicular from Q to the line l. Then ΔPQR is a right angled triangle.

Also PR = $M_1M_2 = OM_2 - OM_1 = x_2 - x_1$
And QR =$QM_2 - RM_2$
= $QM_2 - PM_1$
= $ON_2 - ON_1$
= $y_2 - y_1$

Now $PQ^2 = PR^2 + QR^2$ (Pythagoras theorem)
= $(x_2-x_1)^2 + (y_2-y_1)^2$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

3. Section Formula

Internal Division : If point P divides A(x_1,y_1) and B(x_2,y_2) internally in ratio m_1:m_2:

$$\boxed{P\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)}$$

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two given points on a line l and $R(x, y)$ divide PQ internally in the ratio $m_1 : m_2$.

To find : The coordinates x and y of point R.

Construction : Draw PL, QN and RM perpendiculars to XX' from P, Q and R respectively and L, M and N lie on XX'. Also draw $RT \perp QN$ and $PV \perp QN$.

Method : R divides PQ internally in the ratio $m_1 : m_2$.

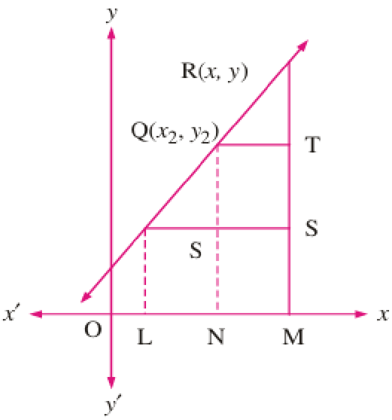
$\Rightarrow$ R lies on PQ and $\dfrac{PR}{RQ}=\dfrac{m_1}{m_2}$

Also, in triangles, RPS and QRT,

$\angle RPS = \angle QRT$ (Corresponding angles as $PS \parallel RT$)

and $\angle RSP = \angle QTR = 90^0$

$\therefore$ $\Delta RPS \sim \Delta QRT$ (AAA similarity)



$\Rightarrow$ $\dfrac{PR}{RQ}=\dfrac{RS}{QT}=\dfrac{PS}{RT}$ ... (i)

Also, $PS = LM = OM - OL = x - x_1$

$RT = MN = ON - OM = x_2 - x$

$RS = RM - SM = y - y_1$

$QT = QN - TN = y_2 - y$.

From (i), we have

$\therefore \dfrac{m_1}{m_2}=\dfrac{x-x_1}{x_2-x}=\dfrac{y-y_1}{y_2-y}$

$\Rightarrow m_1(x_2-x)=m_2(x-x_1)$

and $m_1(y_2-y)=m_2(y-y_1)$

$\Rightarrow$ $x=\dfrac{m_1x_2+m_2x_1}{m_1+m_2}$ and $y=\dfrac{m_1y_2+m_2y_1}{m_1+m_2}$

Thus, the coordinates of R are:

$$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

Coordinates of the mid-point of a line segment

If R is the mid point of PQ, then,

$m_1 = m_2 = 1$ (as R divides PQ in the ratio 1:1

Coordinates of the mid point are $\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$

External Division :

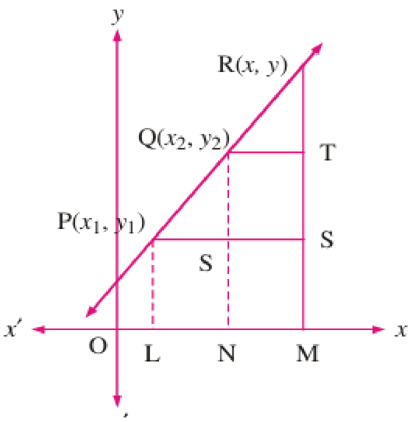
$$P\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$$

Let R divide PQ externally in the ratio $m_1 : m_2$

To find : The coordinates of R.

Construction : Draw PL, QN and RM perpendiculars to XX' from P, Q and R respectively and $PS \perp RM$ and $QT \perp RM$.

Clearly, $\triangle RPS \sim \triangle RQT$.



$$\therefore \frac{RP}{RQ} = \frac{PS}{QT} = \frac{RS}{RT}$$

$$\text{or} \quad \frac{m_1}{m_2} = \frac{x - x_1}{x - x_2} = \frac{y - y_1}{y - y_2}$$

$$\Rightarrow m_1(x - x_2) = m_2(x - x_1)$$

and $m_1(y - y_2) = m_2(y - y_1)$

These give:

$$x = \frac{m_1x_2 - m_2x_1}{m_1 - m_2} \quad and \quad y = \frac{m_1y_2 - m_2y_1}{m_1 - m_2}$$

Hence, the coordinates of the point of external division are

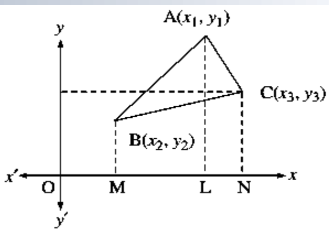
$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$$

Midpoint Formula :

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

4. Area of Triangle

For vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$:



$$\text{Area} = \frac{1}{2}\,|\,x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\,|$$

Important Use : If Area = 0 ⇒ Points are **collinear**.

Let us find the area of a triangle whose

vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$

Draw AL, BM and CN perpendiculars to XX'.

area of $\triangle$ ABC

= Area of trapzium. $BMLA$ + Area of trapzium. ALNC – Area of trapzium. BMNC

$$= \frac{1}{2}(BM + AL)ML + \frac{1}{2}(AL + CN)LN - \frac{1}{2}(BM + CN)MN$$

$$= \frac{1}{2}(y_2 + y_1)(x_1 - x_2) + \frac{1}{2}(y_1 + y_3)(x_3 - x_1) - \frac{1}{2}(y_2 + y_3)(x_3 - x_2)$$

$$= \frac{1}{2}\left[(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + (x_3 y_1 - x_3 y_3)\right]$$

$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

This can be stated in the determinant form as follows :

$$\text{Area of } \Delta\, ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

5. Condition of Collinearity

Three points are collinear if:

$$\text{Area of triangle formed} = 0$$

Or

i.e $\frac{1}{2}[x_1x_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1x_3] = 0$

i.e. $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

In short, we can write this result as $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

Inclination : Angle θ made by a line with positive x-axis (anticlockwise).

6. Inclination and Slope

Special Cases:

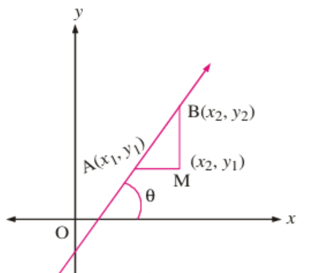
- Horizontal line → m = 0
- Vertical line → slope undefined
- Line equally inclined to axes → m = ±1

7. Slope of Line Joining Two Points

For A(x_1,y_1) and B(x_2,y_2) :

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

If $x_1 = x_2$, slope is undefined.



(a)

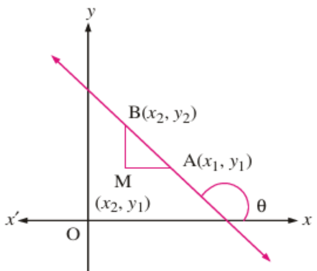
(A) In Fig 13.14 (a), angle of inclination MAB is equal to θ (acute). Consequently.

$$\tan\theta = \tan(\angle MAB) = \frac{MB}{AM} = \frac{y_2 - y_1}{x_2 - x_1}$$

(B) In Fig. 13.14 (b), angle of inclination θ is obtuse, and since θ and $\angle MAB$ are supplementary, consequently,

$$\tan\theta = -\tan(\angle MAB) = -\frac{MB}{MA} = -\frac{y_2 - y_1}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

Hence in both the cases, the slope m of a line through $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by



(b)

8. Parallel and Perpendicular Lines

Parallel Lines : $\boxed{m_1 = m_2}$

Perpendicular Lines : $\boxed{m_1 m_2 = -1}$

SLOPE OF PARALLEL LINES :

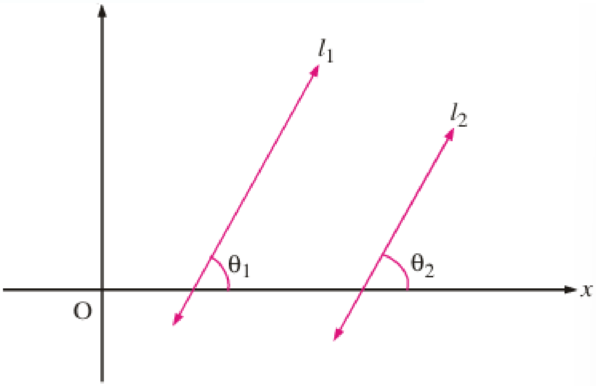
Let l_1, l_2, be two (non-vertical) lines with their slopes m_1 and m_2 respectively.

Let θ_1 and θ_2 be the angles of inclination of these lines respectively.

Case I : Let the lines *l_1 and l_2 be parallel*

Then $\theta_1 = \theta_2 \Rightarrow \tan\theta_1 = \tan\theta_2$

$\Rightarrow m_1 = m_2$



Thus, if two lines are parallel then their slopes are equal.

Case II : Let the lines l_1, and l_2 have equal slopes.

i.e. $m_1 = m_2 \Rightarrow \tan\theta_1 = \tan\theta_2$

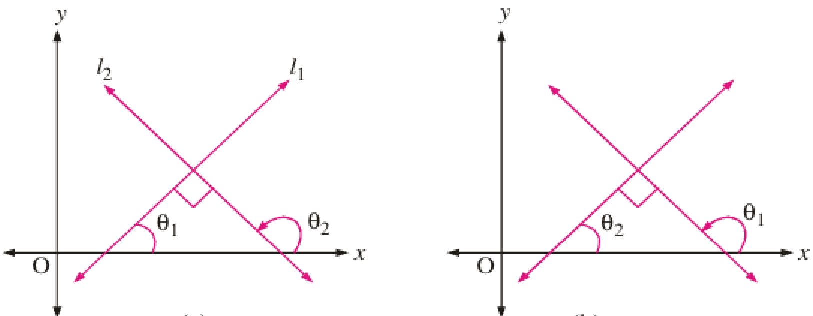
$\Rightarrow \theta_1 = \theta_2 \ (0^\circ \le \theta \le 180^\circ)$

$\Rightarrow l_1 \parallel l_2$

Hence, two (non-vertical) lines are parallel if and only if $m_1 = m_2$

SLOPE OF PERPENDICULAR LINES :

Let l_1 and l_2 be two (non-vertical)lines with their slopes m_1 and m_2 respectively. Also let θ_1 and θ_2 be their inclinations respectively.



(a) (b)

Case-I : Let $l_1 \perp l_2$

$\Rightarrow \theta_2 = 90^0 + \theta_1$ or $\theta_1 = 90^0 + \theta_2$

$\Rightarrow \tan\theta_2 = \tan(90^0 + \theta_1)$ or $\tan\theta_1 = \tan(90^0 + \theta_2)$

$\Rightarrow \tan\theta_2 = -\cot(\theta_1)$ or $\tan\theta_1 = -\cot(\theta_2)$

$\Rightarrow \tan\theta_2 = -\dfrac{1}{\tan\theta_1}$ or $\Rightarrow \tan\theta_1 = -\dfrac{1}{\tan\theta_2}$

$\Rightarrow$ In both the cases, we have

$\tan\theta_1 \tan\theta_2 = -1$

or $m_1 . m_2 = -1$

Thus, if two lines are perpendicular then the product of their slopes is equal to -1.

Case II : Let the two lines l_1 and l_2 be such that the product of their slopes is -1.

i.e. $m_1.m_2 = -1$

$$\Rightarrow \tan\theta_1 \tan\theta_2 = -1$$

$$\Rightarrow \tan\theta_1 = -\frac{1}{\tan\theta_2} = -\cot\theta_2 = \tan\left(90^0 + \theta_2\right)$$

or

$$\tan\theta_2 = \frac{-1}{\tan\theta_1} = -\cot\theta_1 = \tan\left(90 + \theta_1\right)$$

$\Rightarrow$ Either $\theta_1 = 90^0 + \theta_2$ or $\theta_2 = 90^0 + \theta_1$ $\Rightarrow$ In both cases $l_1 \perp l_2$.

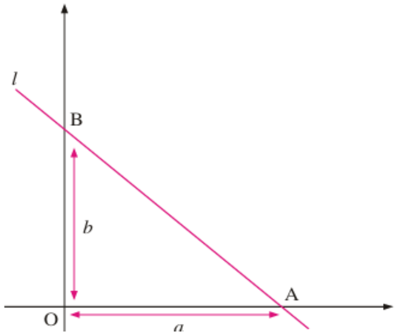
Hence, two (non-vertical) lines are perpendicular if and only if $m_1.m_2 = -1$.

9. Intercepts of a Line

For a line:

- **x-intercept** → Put $y = 0$
- **y-intercept** → Put $x = 0$

If a line cuts x-axis at (a,0) and y-axis at (0,b):



Note: 1. A line which passes through origin makes no intercepts on axes.
2. A horizontal line has no x-intercept and vertical line has no y-intercept.
3. The intercepts on x- axis and y-axis are usually denoted by a and b respectively. But if only y-intercept is considered, then it is usually denoted by c.

10. Angle Between Two Lines

$$\frac{x}{a} + \frac{y}{b} = 1$$

If slopes are m_1 and m_2: $\tan\ \theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ where θ is the acute angle between the lines.

Let l_1 and l_2 be two non vertical and non perpendicualr lines with slopes m_1 and m_2 respectively. Let α_1 and α_2 be the angles subtended by l_1 and l_2 respectively with the positive direction of x-axis. Then $m_1 = \tan\alpha_1$ and $m_2 = \tan\alpha_2$.

From figure 1, we have $\alpha_1 = \alpha_2 + \theta$

$\therefore \quad \theta = \alpha_1 - \alpha_2$

$\Rightarrow \quad \tan\theta = \tan(\alpha_1 - \alpha_2)$

i.e. $$\tan\theta = \frac{\tan\alpha_1 - \tan\alpha_2}{1 + \tan\alpha_1 . \tan\alpha_2}$$

i.e. $$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} \quad ...(1)$$

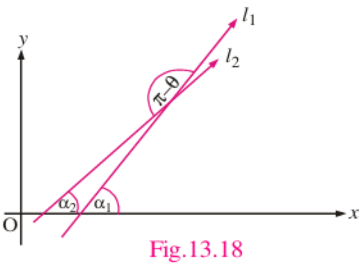


Fig.13.18

As it is clear from the figure that there are two angles θ and $\pi - \theta$ between the lines l_1 and l_2.

We know, $\tan(\pi - \theta) = -\tan\theta$

$$\therefore \quad \tan(\pi - \theta) = -\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$$

Let $\pi - \theta = \phi$

$$\therefore \quad \tan\phi = -\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right) \quad ...(2)$$

- If $\frac{m_1 - m_2}{1 + m_1 m_2}$ is positive then $\tan\theta$ is positive and $\tan\phi$ is negative i.e. θ is acute and ϕ is obtuse.

- If $\frac{m_1 - m_2}{1 + m_1 m_2}$ is negative then $\tan\theta$ is negative and $\tan\phi$ is positive i.e. θ is obtuse and ϕ is acute.

Thus the acute angle (say θ) between lines l_1 and l_2 with slopes m_1 and m_2 respectively is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \text{ where } 1 + m_1 m_2 \neq 0.$$

The obtuse angle (say ϕ) can be found by using the formula $\phi = 180° - \theta$.

Translation (Shifting of Origin)

If origin shifts from O to (h,k):

$$x' = x - h$$

$$y' = y - k$$

or

$$x = x' + h, \qquad y = y' + k$$

New coordinates of point P(x,y):

$$P(x - h,\ y - k)$$